

Derivatives

MS : Put Call Parity Theorem [PCPT]

$$\begin{array}{ccccccc}
 C_0 + P_0 e^{-rT} K & = & P_0 & + & S_0 \\
 \downarrow & & \downarrow & & \downarrow & & \downarrow \\
 \text{Value of} & & \text{Strike} & & \text{Value of} & & \text{Share} \\
 \text{Call} & & \text{Price} & & \text{Put} & & \text{Price} \\
 \text{today} & & & & \text{today} & & \text{today}
 \end{array}$$

(I) Lower Boundary of a Call [LBC]

$$C_0 \geq S_0 - P_0 e^{-rT} K$$

Eg:

$$S_0 = 500$$

$$K = 470$$

$$r = 10\%$$

$$n = 1 \text{ year}$$

Cal. of LBC

$$C_0 \geq S_0 - P_0 e^{-rT} K$$

$$\geq 500 - \frac{470}{(1.1)^1}$$

$$\geq 500 - 427.27$$

$$C_0 \geq 72.73$$

Case 1 : If Actual $C_0 < 72.73$
Let's say $C_0 = 60$

This gives rise to an Arbitrage Opportunity.

$\therefore C_0$ is underpriced, Strategy for Arbitrage:

BUY Call

&

SELL Share

Cal. of Net ₹ Inflow today:

Step 1:	Buy Call	(60)
Step 2:	Sell Share	500
	Net ₹ Inflow	440

Step 3: Invest ₹ 440 today @ 10% for 1 year.

$$MV = 440 (1.1)^1$$

$$= 484$$

Cal. of CF on DD:

Particulars	$S_1 > K$	$S_1 < K$
	Call: ITM	Call: OTM
	$S_1 = 600$	$S_1 = 300$
Step 4: Realise MP of Investment	484	484
Step 5: Buy share @ S_1	(600)	(300)
Step 6: GP on Call	130	0
	$[S_1 - K]$	$[-]$
Arbitrage Gain	14	184

This Strategy will provide Gain [Min. 14 or Higher], irrespective of S_1 ; $\therefore$, Arbitrage!

Due to this arbitrage opportunity, Demand for 'Buy Call & Sell Share' Strategy will increase. Due to this, demand for Call will increase thereby pushing C_0 above ₹60. Once it touches ₹72.73, Arbitrage opp. ceases to exist. This is why, C_0 can't hang around below 72.73. $\therefore$, ₹72.73 is LBC.

Case 2: If Actual $C_0 > 72.73$
Let's say $C_0 = 90$

This gives rise to an 'Arbitrage Opportunity'.

$\therefore C_0$ is over priced, Strategy for Arbitrage:

SELL Call

&

BUY Share

Cal. of Net ₹ Outflow today:

Step 1: Sell Call 90
Step 2: Buy Share (500)
Net ₹ Outflow (410)

Step 3: Borrow ₹ 410 today @ 10% for 1 year.

$$MV = 410 (1.1)^1$$

$$= 451$$

Cal. of CF on DD:

Particulars	$S_1 > K$	$S_1 < K$
	Call: ITM	Call: OTM
	$S_1 = 600$	$S_1 = 300$
Step 4: Pay MV of Borrowing	(451)	(451)
Step 5: Sell share @ S_1	600	300
Step 6: GP on Call	(130)	0
	<u>$[K - S_1]$</u>	<u>$[-]$</u>
Arbitrage Gain	19	(151)

∴, this strategy gives Gain only if $S_1 > K$.

∴, Such Strategy Can't be Called as Arbitrage.

Alternate understanding

Consider foll. 2 Portfolios:

$$P(A) = 1\text{Call} + \text{Cash Equivalent to PV of } K$$

$$P(B) = 1\text{share}$$

Cal. of VCPD on DD:

Particulars	$S_1 > K$ [Call:ITM]	$S_1 < K$ [Call:OTM]
$P(A)$	$[S_1 - K] + K$ $= S_1$	$0 + K$ $= K$
$P(B)$	S_1	S_1

$$\text{If } S_1 > K, P(A) = P(B)$$

$$\text{If } S_1 < K, P(A) > P(B)$$

$\therefore$, Irrespective of S_1 ,

$$P(A) \geq P(B)$$

$$C_0 + \text{PV of } K \geq S_0$$

$$\therefore, C_0 \geq S_0 - \text{PV of } K$$

Q.427 (i) Cal. of Th. Min. Price of a Call [LBC]

$$C_0 \geq S_0 - PV \text{ of } K$$

$$\geq 200 - \frac{180}{(1.1)^1}$$

$$\geq 200 - 163.64$$

$$C_0 \geq 36.36$$

(ii) If $C_0 = 30$

$\therefore C_0$ is underpriced, Strategy for Arbitrage:

Buy Call
↓
Sell Share

Cal. of Net ₹ Inflow today:

Step 1: Buy Call (30)

Step 2: Sell Share 200

Net ₹ Inflow 170

Step 3: Invest ₹ 170 today @ 10% for 1 year.

$$MV = 170 (1.1)^1$$

$$= 187$$

Cal. of CF on DD:

Particulars	$S_1 > K$ Call: ITM	$S_1 < K$ Call: OTM
let's say $S_1 = 250$		$S_1 = 150$
Step 4: Realise MP of Inv+	187	187
Step 5: Buy Share @ S_1	(250)	(150)
Step 6: GP on Call	70 i.e. $250 - 180$	0
Arbitrage Gain	7	37

